Theoretical Description of Proton Radioactivity

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The theory for proton radioactivity from deformed drip-line nuclei is discussed. All experimental data available on decay from ground and isomeric states, and fine structure of odd-even and odd-odd nuclei, are consistently interpreted without free parameters, by this theoretical approach.

§1. Introduction

Recent studies with exotic nuclei lead to the discovery of a new form of radioactivity [1), 2)] in nuclei lying beyond the proton drip-line. Protons can be emitted from the ground state, but decays from isomeric excited states of the parent nucleus, and fine structure for decay to an excited 2^+ rotational state of the daughter nucleus [3)] were also observed.

Proton emission occurs in the region of $50 < Z < 82$, where the Coulomb barrier is very high and the proton can be trapped in a resonance state which will decay by tunnelling through the barrier for almost 80 fm, thus leading to quite narrow decay widths. The measurement of the escape energy of the emitted proton has shown values of the order of 1–2 MeV, which are quite small. This has two important consequences. It shows that the emitter is very close to the proton drip-line, so it is a way of mapping it, and also implies that the proton was in a resonance state very low in the continuum, corresponding essentially to single particle excitations.

From the previous discussion, the theory to describe proton radioactivity that provides the formalism for the evaluation of experimental observables, like decay widths and branching ratios, can be based on the evaluation of single particle resonances in exotic nuclei. The observed emitters range from spherical or quasi spherical nuclei, up to nuclei with large deformations $\beta \approx .3$. For nuclei that are expected to be spherical, since the potential barrier is very large, a WKB calculations can already give a good estimate of the experimental data. [4)] For deformed nuclei, Nilsson resonances should be evaluated [5)] in the deformed nuclear mean field.

The purpose of this work is to show how decay from deformed nuclei can be studied in terms of these resonances. The parent nucleus can be treated in the strong coupling limit of the particle-rotor model, [6)] as a first approach. The inclusion [7)] of Coriolis mixing, requires a proper treatment of the pairing residual interaction, leading to a unified interpretation of all available data on these emitters.

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§2. Adiabatic approach: the strong coupling limit

Since nuclei on the drip-line have a Fermi level very close or even immersed in the continuum, it is possible to consider proton radioactivity in deformed nuclei as decay from a single particle resonance of the unbound core-proton system, and determine the corresponding decay width. The first step is the calculation of Nilsson resonances. The wave function of the decaying proton can be obtained from the exact solution of the Schrödinger equation with a deformed mean field with deformed spin-orbit, imposing outgoing wave boundary conditions, as discussed in Ref. 8).

The dependence on the parameterization of the single particle potential was discussed in Ref. 9), and proved to be of no significance for well established potentials like the ones discussed if Ref. 10), with the exception of the Becchetti and Greenlees potential,[11)] derived from scattering data of lighter nuclei. The deformed spin-orbit term requires special care for a precise solution of the equation. However, it is possible to include it, without approximations,[8)] and it was found to give a non-negligible contribution to the decay width.[9), 12)]

The partial decay width can be determined from the overlap between the initial and final states. Therefore, a nuclear structure model has to be considered in order to determine the wave function of the parent nucleus. The simplest approach is to impose the strong coupling limit,[6)] where the nucleus behaves as a particle plus rotor with infinite moment of inertia.

Within these assumptions, if decay occurs to the ground state, due to angular momentum conservation only the component of the s.p. wave function with the same angular momentum as the ground state of the daughter nucleus contributes, that is, $j_p = J_i = K_i$, and the decay width becomes,

$$\Gamma_{l_p j_p}(r) = \frac{\hbar^2 k}{\mu(j_p + 1/2)} \frac{|u_{l_p j_p}(r)|^2}{|G_{l_p}(kr) + iF_{l_p}(kr)|^2} u_{K_i}^2, \qquad (2\cdot1)$$

where F and G are the regular and irregular Coulomb functions, respectively, and $u_{l_p j_p}$ the component of the wave function with momentum j_p, equal to the spin of the decaying nucleus. The quantity $u_{K_i}^2$ is the probability that the single particle level in the daughter nucleus is empty, evaluated in the BCS approach.

In this case of decay to the ground state, the unique component of the wave-function tested could be very small, but proton radioactivity will be sensitive to such details. Decay to excited states, allow few combinations for $l_p j_p$ according to angular momentum coupling rules, and consequently different components of the parent wave function are then tested. Therefore, a very precise calculation of these states is crucial, and the inclusion of deformed spin-orbit is unavoidable. Similar calculations were done within the coupled-channel Green's function model,[13)] where a formalism in terms of Green's function was developed, leading at this level of approximation, to results very similar to the ones obtained with our method.

The decay width obtained from Eq. (2·1) depends on deformation, and is very sensitive to the wave function of the decaying state. Therefore, if it is able to reproduce the experimental value, will give clear information on the deformation and properties of the decaying state. The method is illustrated in Fig. 1 for the

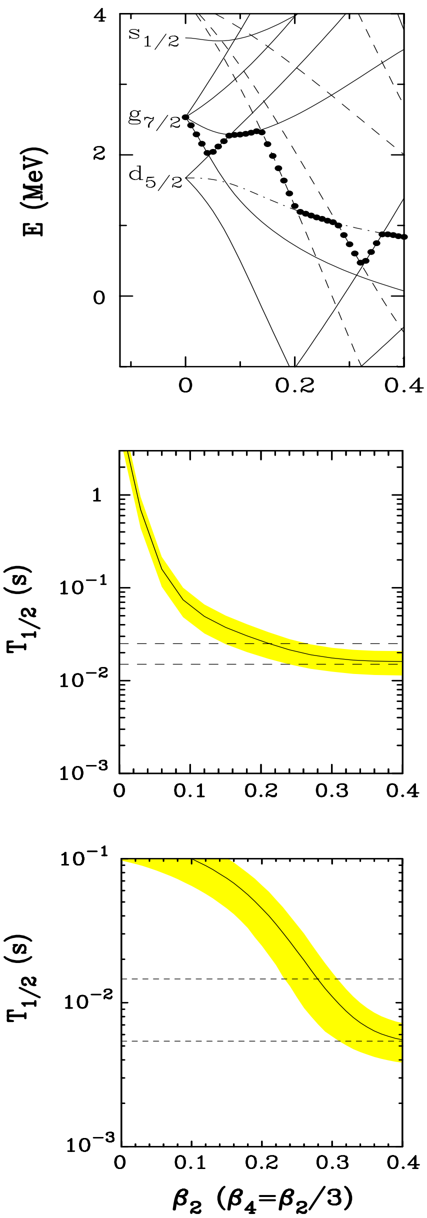

Fig. 1. Decay from ^{117}La. Proton Nilsson levels (upper part). The full circles represent the Fermi surface, the dashed lines the negative parity states, and the dashed-dotted line, the decaying state. A hexadecapole deformation $\beta_4 = \beta_2/3$ was included. Half-life for decay from the ground state $K = 3/2^+$ as a function of β (middle). As in the middle section for the isomeric state $K = 9/2^+$ (lower part). The experimental value[15)] is within the dashed lines.

proton emitter ^{117}La. The state $K = 3/2^+$ reproduces the experimental half-life for a deformation $\beta_2 \approx .2 - .3$, with a small hexadecapole contribution $\beta_4 = .1$, in close agreement with the theoretical predictions of Ref. 14).

The states close to the Fermi surface are the most probable ones for decay to occur. Therefore, the calculation of the half-lives for decay from some of these states, will cross the experimental "area" in a region defined by a certain deformation β. It may happen, that more than one s.p. states can reproduce the experimental decay width, but if isomeric decay or fine structure are measured, the extra experimental data provided by the branching ratios for these processes should be consistently reproduced, thus imposing extra constraints to be fulfilled by the decaying state. In our example of ^{117}La, decay from an isomeric state was also measured,[15)] and it was interpreted with a $9/2^+$ at the same deformation already attributed to the parent nucleus when decay from the $K = 3/2^+$ to the ground was identified as it can be seen in Fig. 1. The example of ^{131}Eu discussed in Ref. 16) shows how fine structure data helped to identify the decaying state and the deformation of the emitter.

Emission from deformed systems with an odd number of protons and neutrons can be discussed in a similar fashion.[17)] The decaying nucleus is described by a wave function of two particles-plus-rotor in the strong coupling limit. Therefore, in contrast with decay to ground state of odd-even nuclei where the proton is forced to escape with a specific angular momentum, many channels will be open due to the angular momentum coupling of the proton and daughter nucleus, $\vec{J}_d + \vec{j}_p$, giving the total width for decay as a sum of partial widths, for all possible channels with quantum numbers allowed by parity and momentum conservation. This dependence on the quantum numbers of the unpaired neutron, gives to the neutron the important role of "influential spectator" contributing significantly with its angular momentum to the decay. Moreover, the identification of the proton state by the radioactive decay, will lead to the determination of the neutron s.p. level in the emitter, important nuclear structure information impossible to obtain otherwise. It should also be noticed that in these calculations of odd-odd emitters, the Nilsson state of the proton is the same as the one in the neighbour odd-even nucleus, with the same deformation.

We have applied [5),16)–18)] our model to all measured deformed proton emitters including isomeric decays. The experimental half-lives are perfectly reproduced by a specific state, with defined quantum numbers and deformation, thus leading to unambiguous assignments of the angular momentum of the decaying states and also supporting previous predictions [14)] on nuclear structure properties of the emitter. Extra experimental information provided by isomeric decay observed in ^{117}La, ^{141}Ho and ^{151}Lu, and fine structure in ^{131}Eu can also be successfully accounted by the model. The experimental half-lives for decay from the excited states were reproduced in a consistent way with the same deformation that describes ground state emission.

§3. The non-adiabatic quasi-particle approach: contributions from Coriolis mixing and pairing residual interaction

As we have discussed in the previous section, calculations within the strong coupling limit were able to reproduce the experimental results. According to this model, the daughter nucleus has an infinite moment of inertia, and the rotational spectrum collapses into the ground state. Considering a finite moment of inertia, the Hamiltonian of the decaying nucleus can be decomposed into a term acting on the degrees of freedom of the rotor, a recoil term acting on the coordinates of the valence proton, and a term representing a purely kinematic coupling between the degrees of freedom of both, known as the Coriolis coupling. Therefore, the wave functions of the rotor are modified with respect to the adiabatic approach, since its rotational spectrum is included, and another interaction is acting on the nucleus, the Coriolis force.

The effect of a finite moment of inertia of the daughter nucleus on proton decay, was studied within the non-adiabatic coupled channel,[19)] and coupled-channel Green's function [12)] methods, but the excellent agreement with experiment found in the adiabatic context was lost. The results differ by factors of three or four from the experiment, and even the branching ratio for fine structure decay is not reproduced.[19),12),20)] This result is surprising, since calculations that include the Coriolis mixing should undoubtedly be better. The use in the calculations of Refs. 19) and 20) of a spherical spin-orbit mean field is a strong handicap of their model, and is responsible for the large deviations observed in ^{131}Eu and ^{117}La, nuclei with low angular momentum where the Coriolis coupling should be small. However, the strange behaviour found also in the calculation of Ref. 12) which includes deformed spin-orbit, for the decay of ^{141}Ho needs an explanation.

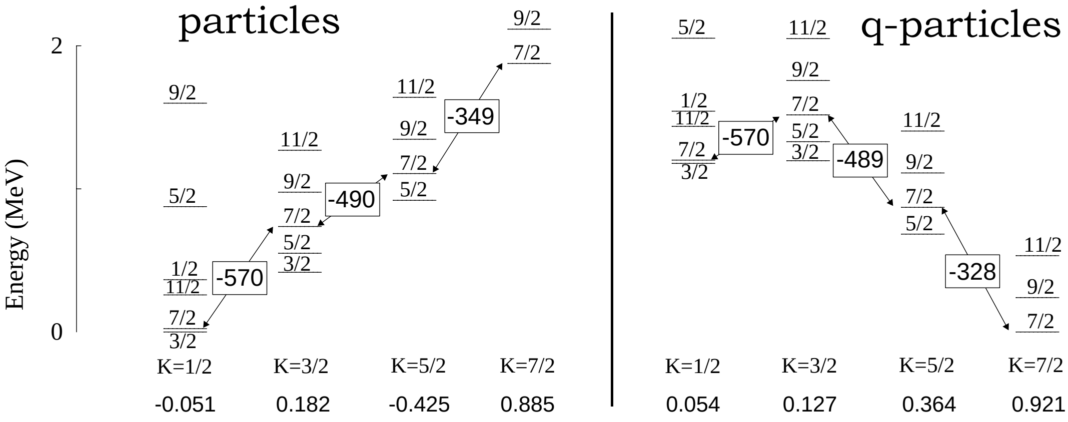

Fig. 2. The left section of the figure represents the level scheme for particle states at energies equal to the diagonal matrix elements of the nuclear Hamiltonian. The numbers in boxes are the off-diagonal matrix elements of the Coriolis force (in keV). States with same angular momentum are connected. The right section as in the left for quasi-particles. The numbers in the bottom row are the components of the wave function of the $7/2^-$ decaying state of ^{141}Ho.

Decay rates in deformed nuclei, are extremely sensitive to small components of the wave function. The Coriolis interaction mixes different Nilsson wave functions, and can be responsible for strong changes in the decay widths. However, the residual pairing interaction can modify this mixing of states, an effect not considered in the calculations of Refs. 19), 12) and 20). We have included [7] beside the Coriolis mixing, the pairing residual interaction in the BCS approach. The mixing of states is modified by the residual interaction and transformed trough a Bogoliubov transformation into a mixing between quasi-particle states instead of particle ones, as was used in Refs. 19), 12) and 20).

Let us consider for example the decay of the $7/2^-$ ground state of ^{141}Ho to the ground and first 2^+ excited states in ^{140}Dy. This was the most strange case in the calculation of Ref. 12). Since it involves the spherical $h11/2$ state, which has a very high angular momentum, a strong Coriolis force is expected. Including the Coriolis interaction, the decay width to the ground state decreases drastically, leading to an increase of the branching ratio. With the residual pairing interaction the decay width to the ground state increases, leaving the width for decay to the excited state unchanged, and reducing the branching ratio.

This can be understood from the analysis of the level scheme corresponding to the basis states displayed in Fig. 2, at energies equal to the diagonal matrix elements of the Hamiltonian of the nucleus with the Coriolis interaction for particles, and after the transformation to quasi-particles. The difference between both representations, is an inversion of the level ordering, while the off-diagonal matrix elements are practically unchanged. After diagonalization, the wavefunction that describes the decaying nucleus corresponds, in the particle case, to the highest state in energy, while in the quasi-particles to the lowest one, as it should be. This inversion implies a change of sign of the wave function components, leading to an interference between these components, in the calculation of the decay width. For particles the interference is destructive and the width decreases, while it turns out to be constructive for quasi-particles. The width is enhanced, and the adiabatic results are recovered.

Such non-adiabatic treatment of the Coriolis coupling, brings back the perfect agreement with data observed in the strong coupling limit. Therefore, the previous disagreement between the calculation with Coriolis[19),12),20)] and the experimental data or between calculations with Coriolis and the ones in the strong coupling limit,[5)] were simply due to an inadequate treatment of the residual pairing interaction.

§4. Conclusions

We have presented a unified model to describe proton radioactivity from deformed nuclei. Decay is understood as decay from single particle Nilsson resonances that are evaluated exactly for single particle potentials that fit large sets of data on nuclear properties. The rotational spectra of the daughter nucleus, and the pairing residual interaction in the BCS approach, are taken into account, leading to a treatment of the Coriolis coupling in terms of quasi-particles. All available experimental data on even-odd and odd-odd deformed proton emitters from the ground and isomeric states and fine structure, are accurately and consistently reproduced by the model.

The calculation provides valuable nuclear structure information on deformation and angular momentum J of the decaying nucleus, and also on the state of the unpaired neutron in decay from odd-odd nuclei, thus giving unambiguous assignments to the decaying states.

Proton radioactivity provides a unique tool to access nuclear structure properties of nuclei far away from the stability domain.

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