Nuclear Fusion in the Sun

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We discuss the possibility of studying nuclear reactions in the sun by means of solar neutrinos and helioseismology. In particular we review the observational information which is available on the energy source of the sun, cross sections and screening in the solar interior.

§1. Introduction

Some forty years ago John Bahcall and Raymond Davis started an exploration of the Sun by means of neutrinos. 1) Their journey had a long detour, originating the so called solar neutrino puzzle. After forty years, thanks to the solar neutrino experiments and to the Kamland reactor neutrino experiment (see Ref. 2) for a list of references) the solar neutrino puzzle has been understood in terms of neutrino oscilllations. Clearly, there is still a long road for a complete description of the neutrino mass matrix, however now that we know the fate of neutrinos we can go back to the original program started by Davis and Bahcall and ask what can be learnt on the Sun from the study of solar neutrinos.

Another probe of the deep solar interior is provided by helioseismology (see e.g. Refs. 3) and 4)). The highly precise measurements of frequencies and the tremendous number of measured lines enable us to extract both the properties of the convective envelope (depth, helium abundance) and the sound speed along the solar profile with high accuracy.

The production of solar neutrinos is mainly sensitive to the solar temperature. On the other hand, from helioseismic observations one cannot determine directly the temperature of the solar interior, as one cannot determine the temperature of a gas from the knowledge of the sound speed unless the chemical composition is known. Neutrino and helioseismic information are thus complementary.

We shall review the information on nuclear fusion in the sun provided by neutrinos and by helioseismology. In particular we shall address the following questions:

- What is the energy source powering the sun?
- What do we know about nuclear cross sections in the solar interior?
- What can be said about screening of nuclear reactions in the sun?

§2. The ^{8}B neutrino flux

Among the various components of the solar neutrino flux, the ^{8}B neutrino flux is particularly important, since it is now a measured quantity. In fact, by combining the

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final Super-Kamiokande (SK) data and the latest SNO charged and neutral current fluxes one obtains the following estimate of the the total active neutrino flux from ^{8}B decay $\Phi_{\rm B} = \Phi(\nu_e + \nu_\mu + \nu_\tau)$:[5)]

$$\Phi_{\rm B} = 5.5\,(1 \pm 7\%)\,10^6 \ {\rm cm}^{-2}{\rm s}^{-1} \quad (1\sigma)\,, \tag{2·1}$$

in good agreement with the predictions of recent Standard Solar Model (SSM) calculations (see e.g. Ref. 4)). The 7% accuracy, which is already smaller than SSM uncertainty, will be improved in the next few years, as a consequence of higher statistics and better experimental techniques.

The ^{8}B neutrino flux depends on several nuclear physics and astrophysical inputs. Scaling laws describe the variation of $\Phi_{\rm B}$ when the input parameters are slightly changed from the SSM value. One has:

$$\begin{aligned}\Phi_{\rm B} = \Phi_{\rm B}^{\rm SSM}\, s_{33}^{-0.43}\, s_{34}^{0.84}\, s_{17}^{-1}\, s_{e7}^{-1}\, s_{11}^{-2.7} \\ \cdot\ lum^{7.2}\, comp^{1.4}\, opa^{2.6}\, age^{1.4}\, dif^{0.34}\,,\end{aligned} \tag{2·2}$$

where for each parameter $x = X/X^{\rm SSM}$. The first line contains the nuclear physics parameters (S_{ij} are the astrophysical factors at zero energy for nuclear reactions $i + j$). The second line groups the astrophysical inputs: $lum \equiv (L/L_\odot)$ expresses the sensitivity to the solar luminosity; $comp \equiv (Z/X)/(Z/X)^{SSM}$ accounts for the metal content of the solar photosphere; $age \equiv (t/t_\odot)$ expresses the sensitivity to the solar age; opa and dif are uniform scaling parameters with respect to the opacity tables and the diffusion coefficients used in SSM calculations.

We clearly understand, from Eq. (2·2), that one can learn astrophysics from the ^{8}B neutrino flux determination only if nuclear physics is known well enough.

In the last few years there has been a significant progress in the experimental study of low energy nuclear reactions (see Ref. 2) for a complete list of references). In particular, the LUNA experiment[6)] has measured the $^3{\rm He} +^3 {\rm He} \to^4 {\rm He} + 2p$ down to solar energies, obtaining a determination of S_{33} with 6% accuracy, which translates into a 3% uncertainty in the prediction of $\Phi_{\rm B}$.

Concerning the reaction $^7{\rm Be} + p \to^8 {\rm B} + \gamma$, until a few years ago the uncertainty on S_{17} was at the level of 10–15%. Several new experiments were performed in the last few years. Quite recently Junghans et al.[7)] presented a new measurement, $S_{17} = 22.1 \pm 0.6\,({\rm expt}) \pm 0.6\,({\rm theor})$ eVb. In addition, by comparing the results of all "modern" direct experiments and by using the same theoretical curve in fitting the data, they obtained as "best value" $S_{17} = 21.4 \pm 0.5\,({\rm expt}) + 0.6\,({\rm theor})\,$eVb. The low-energy global fit is dominated by the data of Ref. 7), all other "modern" direct experiments yielding somehow lower S_{17} values.[8)–11)] Indirect methods for determining S_{17} also suggest a somehow smaller value.[12)] In conclusion, it looks that a 5% accuracy is a reasonable estimate of the present errors on S_{17}, which translates into a 5% uncertainty on $\Phi_{\rm B}$.

The astrophysical factors S_{e7} and S_{11} of reactions $^7{\rm Be}{+}e^- \to^7 {\rm Li}{+}\nu_e$ and $p{+}p \to d + e^+ + \nu_e$ are both known with 2% accuracy, which translates into contributions to the $\Phi_{\rm B}$ uncertainty equal to 2% and 5% respectively. Finally, the astrophysical factor S_{34} of $^3{\rm He} +^4 {\rm He} \to^7 {\rm Be} + \gamma$ is determined with a 9% accuracy, which gives a 8% contribution to $\Phi_{\rm B}$ theoretical error.

In conclusion, combining in quadrature the above contributions, one sees that the global nuclear uncertainty on $\Phi_{\rm B}$ is at the level of 11%, largely due to the errors in S_{34} astrophysical factor. In this respect, the planned new measurement of ^{3}He$+^4$He cross section by LUNA at Gran Sasso is most important.

§3. The central solar temperature

As well known Boron neutrinos can be used as a solar thermometer, since the produced flux depends on a high ($\simeq 20$) power of the temperature near the solar center T.[13),14)] It is time to rediscuss this possibility, since now the boron flux is a measured quantity.

We remind that T is not an independent quantity, its value being the result of the physical and chemical properties of the star. Actually, the various inputs to $\Phi_{\rm B}$ in Eq. (2·2) can be grouped according to their effect on T. All nuclear inputs but S_{11} only determine the weight of the different branches ppI/ppII/ppIII without changing solar structure and temperature. On the other hand, to a large extent the effect of the others can be reabsorbed into a variation of the central solar temperature, almost independently on the way we use to vary it (see Ref. 2) and references therein).

The agreement between the theoretical and the experimental determination of $\Phi_{\rm B}$ clearly indicates that SSMs correctly predict the solar temperature T in the region where ^{8}B neutrinos are produced. We have thus $T \simeq T_{\rm SSM} = 1.57 \cdot 10^7$ K. The present experimental uncertainty on $\Phi_{\rm B}$ (7%) and the errors on nuclear physics parameters yield:

$$\Delta T/T = 0.6\% \,, \tag{3·1}$$

where the main uncertainty arises from S_{34}. In other words, a crucial prediction of SSM has been verified with neutrinos with an accuracy better than 1%.

§4. The energy source of the Sun

According to our understanding of the Sun, most of its power originates from the pp-chain, with a minor contribution ($\approx 1\%$) from the CNO cycle. Although this is theoretically well grounded, an experimental verification is clearly welcome.

The important underlying questions are: is the Sun fully powered by nuclear reactions? Are there additional energy losses, beyond photons and neutrinos?

The idea that the Sun shines because of nuclear fusion reactions can be tested accurately by comparing the observed photon luminosity of the Sun $L_\odot(\gamma)$ with the luminosity inferred from measurements of solar neutrino fluxes, $L_\odot(\nu)$. In fact for each fusion of four proton into a Helium nucleus

$$4p + 2e^- \rightarrow ^4{\rm He} + 2\nu_e \,, \tag{4·1}$$

an energy $Q = 26.73$ MeV is released together with two neutrinos. If one determines from experiments the total neutrino production rate one is also determining the energy production rate in the Sun by means of (4·1) (see §2.1 of Ref. 15)).

Bahcall and Pena-Garay[16)] performed a global analysis of all the available solar and reactor data to determine the allowed range for $L_\odot(\nu)$. Their result is:

$$\frac{L_\odot(\nu) - L_\odot(\gamma)}{L_\odot(\gamma)} = 0.4^{+0.2}_{-0.3} \qquad (1\sigma)\,. \tag{4·2}$$

At 1σ the luminosity of the Sun as inferred from neutrinos is thus determined to about 20%.

A ^{7}Be solar neutrino experiment accurate to 5% could improve this determination to about 13%. The global combination of a ^{7}Be experiment, plus a p-p experiment, plus the existing solar data and three years of KamLAND would make possible a really precise determination of the solar energy produced by nuclear reactions (see Ref. 16)).

§5. Helioseismology and nuclear reactions

Another probe of the deep soalr interior is provided by helioseismology (see e.g. Refs. 3) and 4)). The highly precise measurements of frequencies and the tremendous number of measured lines enable us to extract the sound speed along the solar profile with an accuracy of about 0.15% in a large portion of the Sun. This accuracy degrades to about 1% near the center (see Fig. 1). Recent standard solar model calculations, including element diffusion and using updated opacities and accurate equations of state, are well in agreeement with helioseismic data (see Fig. 1 and Ref. 3)).

Solar models are built by using stellar evolutionary codes which include specific expressions for the nuclear reaction rates. If these latter are changed, the resulting models will be different. Changes can be constrained by requiring that agreeement with helioseismic observations is not spoiled.

In this section we shall consider helioseismic implications on nuclear reaction cross sections and screening in the sun.

5.1. *The $p + p \to d + e^+ + \nu_e$ reaction in the Sun*

The rate of the initial reaction in the pp chain is too low to be directly measured in the laboratory (even in the solar center this rate is extremely small, of the order of 10^{-10} yr^{-1} consistently with the solar age) and it can be determined only by using the theory of low energy weak interactions, together with the measured properties of the deuteron and of the proton-proton scattering. In terms of the astrophysical factor, S(E), what really matters is its zero energy value, which for brevity will be indicated simply as S_{pp}. While we refer to Refs. 17), 18) and 15) for updated reviews, we remark that the calculated values are all in the range (3.89–4.21) 10^{-25} MeV b, i.e. they differ from their mean by no more than 3%. In summary, as input of Standard Solar Model (SSM) calculations, one takes:[18)]

$$S_{pp}^{SSM} = 3.89 \cdot 10^{-25}(1 \pm 0.01)\ \text{MeV b}\,. \tag{5·1}$$

Although some warning is in order as to the meaning of the quoted error, one may conclude that well known physics determines S_{pp} to the level of few per cent or even better.

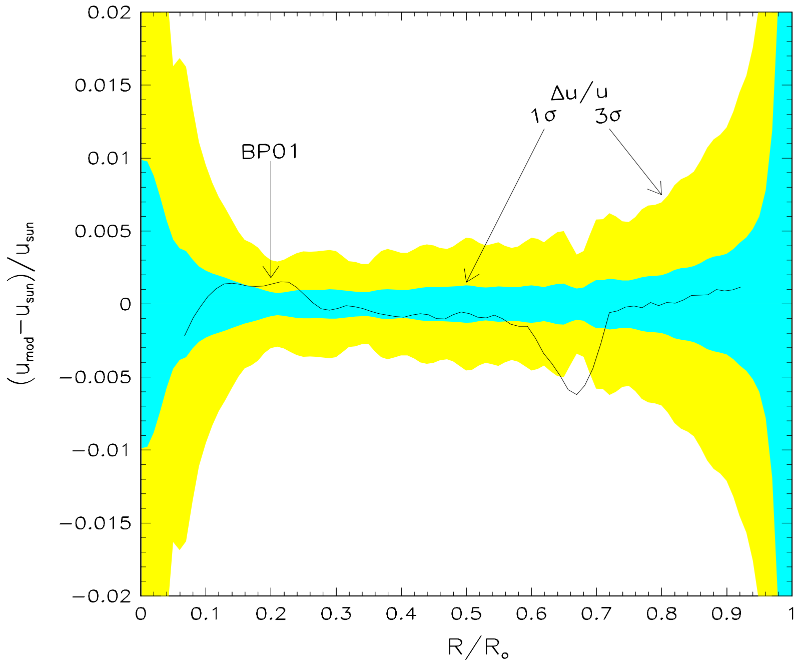

Fig. 1. The dark (light) shaded area corresponds to the 1σ (3σ) uncertainty on helioseismic determination of squared isothermal sound speed $U = P/\rho$.[3] The relative difference between the SSM prediction[4] and the helioseismic data is also shown (thin line).

On the other hand, we remind that only theoretical estimates of S_{pp} are available and observational information would be welcome. In this respect, it is interesting to determine the range of S-values which are acceptable in comparison with helioseismology.[19]

One can understand the effect of changing S_{pp}, at least qualitatively. Since the total fusion rate is fixed by the observed solar luminosity, a value of S_{pp} larger (smaller) than S_{pp}^{SSM} implies smaller (larger) temperature in the solar interior. Since the molecular weight is essentially fixed by the Sun's history, the sound speed in the energy production region has to decrease (increase). This is shown is Fig. 2, from which we derive that at 1σ (3σ) S_{pp} cannot be altered from S_{pp}^{SSM} by more than 2% (6%). We remark that this constraint — comparable to the theoretical uncertainty — relies on observational data.

5.2. *Screening of nuclear reactions in the Sun*

The study of screened nuclear reaction rates was started with the pioneering work of Salpeter[20] who discussed both the extreme cases of "weak" and "strong" screening, providing suitable expressions for the screening factors

$$f_{ij} = \langle \sigma v \rangle_{ij,plasma} / \langle \sigma v \rangle_{ij,bare} \,. \qquad (5{\cdot}2)$$

The solar core is not far from the weak screening case, however it does not satisfy the usual conditions under which the weak screening approximation holds. This is the

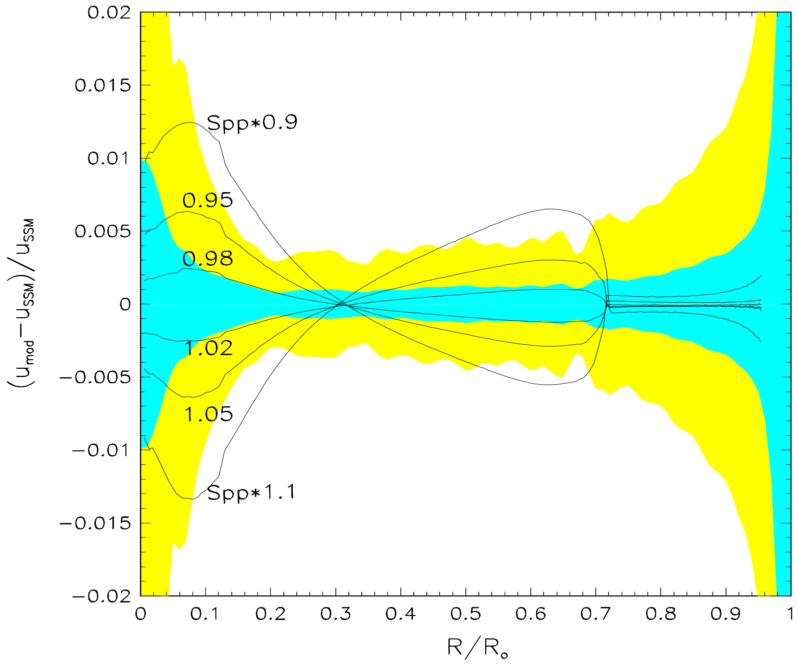

Fig. 2. Comparison of solar models with different values of S_{pp} with the SSM model for $u = P/\rho$, we show the fractional differences, (model - SSM)/SSM. The "statistical" and "conservative" helioseismic uncertainty [3] correspond to the dark and light areas.

reason why the problem has been investigated by several authors (see e.g. Refs. 21) −26)).

Gruzinov and Bahcall [27] calculated the electron density in the vicinity of fusing nuclei using the partial differential equation for the density matrix that is derived in quantum statistical mechanics. Their numerical result agrees, within small uncertainties, with Salpeter's weak screening formula. Furthermore, Bahcall et al. [28] recently provided several arguments that demonstrate the validity of the Salpeter formula near the solar center with insignificant errors.

The conclusions of Gruzinov and Bahcall [27] are not unanimously accepted. According to Shaviv and Shaviv, [29] the weak screening formula does not hold in the Sun. Some nuclear reactions are enhanched by the surrounding plasma whereas some others are suppressed. According to Tsytovitch and Bornatici [30), 31)] a kinetic description of collective plasma effects results in a decrease of the thermonuclear reaction rates in contrast to Salpeter's enhancement.

If one uses different formulas for the screening factors f_{ij} one obtains different nuclear reaction rates and thus different solar models. Most important is the screening factor of the $p + p$ reaction, which governs the energy production rate. Clearly changes f_{pp} have similar effect as changes of S_{pp}.

The main purpose of Ref. 32) is to test the screening models by means of helioseismology. In Ref. 32) several solar models, corresponding to different screening factors, have been built and the results are compared with helioseismic data (see

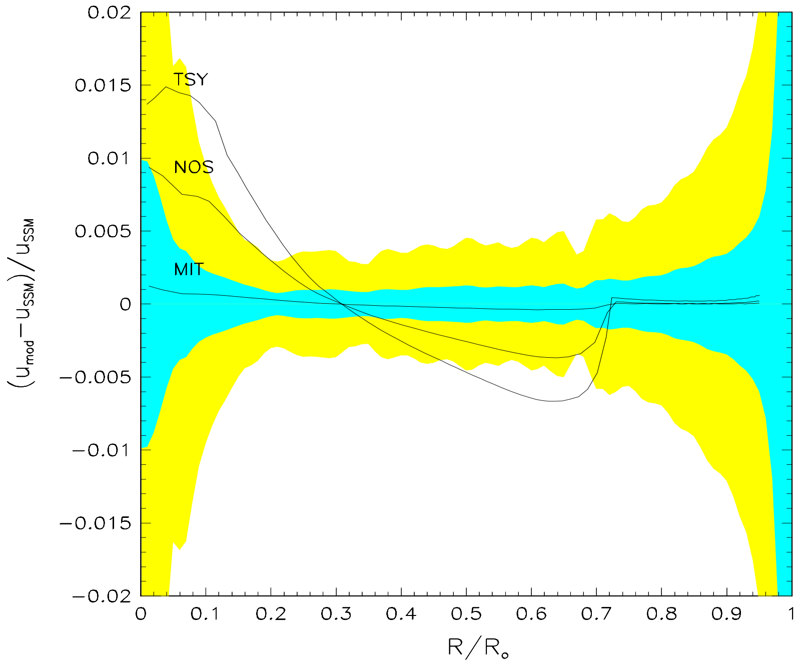

Fig. 3. Comparison of different screening models with the SSM model for $u = P/\rho$, we show the fractional differences, (model - SSM)/SSM. We show the Mitler (MIT),[23] no screening (NOS) and Tsytovitch (TSY)[30] models. The "statistical" and "conservative" helioseismic uncertainty[3] correspond to the dark and light areas.

Fig. 3). We note that:

i) the existence of a screening effect can be proved by means of helioseismology, since the no-screening model (NOS) is excluded at 3σ.

ii) The weak screening approximation, used in SSM calculation, agrees with helioseismic data.

iii) The anti-screening effect (TSY) is excluded by helioseismology.

§6. Concluding remarks

In conclusion we would like to remark the following points:

- Solar neutrinos are becoming an important tool for studying the solar interior and fundamental physics.
- Better determinations of S_{34} (and $S_{1,14}$) are needed for fully exploiting the physics potential of solar neutrinos.
- All this brings towards answering fundamental questions: is the Sun fully powered by nuclear reactions? is the Sun emitting something else beyond photons and neutrinos?

References

1) J. N. Bahcall, Phys. Rev. Lett. **12** (1964), 300.
R. Davis, Phys. Rev. Lett. **12** (1964), 303.
2) G. Fiorentini and B. Ricci, astro-ph/0310753.
3) W. A. Dziembowski et al., Astropart. Phys. **7** (1997), 77.
4) J. N. Bahcall, M. H. Pinsonneault and S. Basu, Astrophys. J. **555** (2001), 990.
5) G. Fogli et al., hep-ph/0309100.
6) LUNA coll., Phys. Rev. C **57** (1998), 2700.
7) A. R. Junghans et al., nucl-ex/0308003.
8) M. Hass et al., Phys. Lett. B **462** (1999), 237.
9) F. Hammache et al., Phys. Rev. Lett. **86** (2001), 219.
10) F. Strieder et al., Nucl. Phys. A **696** (2001), 219.
11) L. T. Baby et al. Phys. Rev. Lett. **90** (2003), 022501.
12) B. Davids and S. Typel, Phys. Rev. C **68** (2003), 045802.
13) V. Castellani et al., Phys. Rev. D **50** (1994), 4749.
14) J. N. Bahcall and R. K. Ulmer, Phys. Rev. D **53** (1996), 4202.
15) V. Castellani et al., Phys. Rep. **281** (1997), 310.
16) J. N. Bahcall and C. Pena-Garay, astro-ph/0305159.
17) J. N. Bahcall and M. H. Pinsonneault, Rev. Mod. Phys. **64** (1992), 885.
18) M. Kamionkowski and J. N. Bahcall, Astrophys. J. **420** (1994), 884.
19) S. Degl'Innocenti, G. Fiorentini and B. Ricci, Phys. Lett. B **416** (1998), 365.
20) E. E. Salpeter, Austr. J. Phys. **7** (1954), 373.
21) H. C. Graboske, H. E. DeWitt, A. S. Grossman and M. S. Cooper, Astrophys. J. **181** (1973), 457.
22) C. Carraro, A. Schäfer and S. E. Koonin, Astrophys. J. **331** (1988), 565.
23) H. E. Mitler, Astrophys. J. **212** (1977), 513.
24) H. Dzitko, S. Turck-Chièze, P. Delbourgo-Salvador and C. Lagrange, to appear in Astrophys. J.
25) B. Ricci, S. Degl'Innocenti and F. Fiorentini, Phys. Rev. C **52** (1995), 1095.
26) T. E. Liolios, Phys. Rev. C **61** (2000), 055802; astro-ph/0002064.
27) A. V. Gruzinov and J. N. Bahcall, Astrophys. J. **504** (1998), 996.
28) J. N. Bahcall et al., astro-ph/0010055.
29) G. Shaviv and N. J. Shaviv, Astrophys. J. **529** (2000), 1054.
G. Shaviv, astro-ph/0010152.
30) V. N. Tsytovich, Astron. Astrophys. **356** (2000), L57.
31) V. N. Tsytovich and M. Bornatici, to appear in Plasma Phys. Rep. (2000).
32) G. Fiorentini, B. Ricci and F. L. Villante, Phys. Lett. B **503** (2001), 121.