Macroscopic Quantum Systems and the Quantum Theory of Measurement

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This paper discusses the question: How far do experiments on the so-called "macroscopic quantum systems" such as superfluids and superconductors test the hypothesis that the linear Schrödinger equation may be extrapolated to arbitrarily complex systems? It is shown that the familiar "macroscopic quantum phenomena" such as flux quantization and the Josephson effect are irrelevant in this context, because they correspond to states having a very small value of a certain critical property (christened "disconnectivity") while the states important for a discussion of the quantum theory of measurement have a very high value of this property. Various possibilities for verifying experimentally the existence of such states are discussed, with the conclusion that the most promising is probably the observation of quantum tunnelling between states with macroscopically different properties. It is shown that because of their very high "quantum purity" and consequent very low dissipation at low temperatures, superconducting systems (in particular SQUID rings) offer good prospects for such an observation.

§ 1. Introduction

It is a great pleasure to dedicate this paper to Professor Ryogo Kubo on the occasion of his sixtieth birthday, and to wish him many more happy and productive years of activity in physics.

This paper represents a first tentative step into an area which I believe is of fundamental interest but fraught with great conceptual difficulties, not all of which it claims to resolve; it is in no way intended to be definitive.

The question I want to discuss is: What experimental evidence do we have that quantum mechanics is valid at the macroscopic level? In particular, do the so-called "macroscopic quantum phenomena" which are actually observed in superconductors and superfluids constitute such evidence? If not, are there other ways in which we can exploit many-body systems in general, and superfluids in particular, to answer the question?

In one sense the answer to our question is rather obvious and not very interesting. There clearly *is* a sense in which many-body systems afford very strong experimental evidence that quantum-mechanical effects are not confined to single atoms or to atomic scales of length and time. For example, the Debye prediction for the low-temperature specific heat of an insulating solid contains the quantum constant of action h, and experimental confirmation of it therefore offers at least circumstantial evidence that collective motions over the scale of many atomic spacings are as subject to the laws of quantum mechanics as those on an atomic scale. More spectacularly, effects such as circulation quantization in superfluid helium or flux quantization in superconductors indicate that quantum coherence effects can operate over distance scales of the order of millimeters, while the Josephson effect shows (among other things) that the purely quantum phenomenon of tunnelling through a potential barrier can produce a current of macroscopic magnitude. Perhaps most spectacularly of all, the Aharonov-Bohm-Mercereau effect[1),2)] shows that the characteristically quantum effect sometimes called the "physical reality of the vector potential" is reflected in the behaviour of macroscopic currents. But none of these effects really extend our experimental evidence for quantum mechanics in qualitative way; what they show in essence is that atoms in large assemblies satisfy the laws of quantum mechanics in much the same way as they do in isolation, and that sometimes (generally because of Bose condensation or the analogous phenomenon (Cooper pairing) in Fermi systems) a macroscopic number of atoms can behave in phase so as to produce macroscopic results.

However, there is a much more subtle and interesting sense in which the question can be interpreted. To motivate this interpretation it is necessary to recall one of the most famous paradoxes in the foundations of quantum mechanics, the so-called Schrödinger's Cat paradox. I shall review the line of argument which leads to the paradox in the next section; for present purposes it is sufficient to recall that it essentially consists in the conclusion that a *macroscopic* object (in the original version, a cat) may be in a linear superposition of states corresponding to macroscopically different behaviour, provided only that it is not "observed". Once an observation or "measurement" is made, however, the system immediately collapses into a state with definite macroscopic properties. Now whatever one's reaction to the paradox, it is clear (cf. next section) that it only arises at all because one has implicitly assumed that the linear laws of quantum mechanics, in particular the superposition principle, apply to the description of any physical system, even when it is of macroscopic dimensions and complexity. The question then arises whether there is any experimental evidence for this assumption: In particular, is there actually any evidence that macroscopic systems can under appropriate conditions be in quantum states which are linear superpositions of states with different macroscopic properties? That is the question to which this paper is devoted.

The plan of the paper is as follows. In the next section I briefly review the Schrödinger's Cat paradox, paying particular attention to the implicit assumptions involved in its formulation. In § 3 I show that the quantum states important in the discussion of the paradox are characterized by a very high value of a certain (qualitatively defined) property, the "disconnectivity"; by contrast, the states necessary to explain the so-called "macroscopic quantum phenomena" in superfluids and superconductors have only low disconnectivity, so that they are irrelevant to our question in the sense intended. In § 4 I discuss in general terms the problem of establishing experimentally the existence in nature of high-disconnectivity states, reaching the conclusion that the most promising area to look is phenomena where quantum tunnelling plays an essential role. Finally, in § 5 I discuss the problem of observing both quantum tunnelling and the quantum coherence phenomena related to it in macroscopic systems, with special reference to the case of flux tunnelling in a SQUID, which at present seems the most promising candidate. Section 6 is a brief conclusion.

§ 2. The Cat paradox

Of all the paradoxes which beset the foundations of quantum mechanics, perhaps the most worrying and intractable is the one first formulated by Schrödinger[3] in 1936 and colloquially known as the Schrödinger's Cat paradox. Here I shall simply outline briefly the main features which are of relevance to this paper; for a more complete discussion see e.g. Ref. 4), part IV.

Consider a microsystem*) whose state can be described in quantum mechanics in terms of a Hilbert space spanned by a complete orthonormal set of eigenvectors $\{\varphi_i\}$ of some observable A with eigenvalues a_i. (When we need a concrete example, it is helpful to think of a particle of spin $\frac{1}{2}$, in which case the 2-dimensional Hilbert space is spanned by the eigenvectors $|\uparrow\rangle$, $|\downarrow\rangle$ corresponding to the eigenvalues $\pm\frac{1}{2}\hbar$ of the spin projection operator S_z). A general "pure state" of the system in the sense of von Neumann is described by the normalized wave function

$$\psi=\sum_i c_i\varphi_i, \quad \sum_i |c_i|^2=1. \tag{2·1}$$

According to the standard quantum measurement axioms as presented in most textbooks, measurement of the physical observable A on a system described by the wave function (2·1) yields the result a_i with probability $|c_i|^2$, and once the measurement has been carried out the wave function is simply φ_i. It is therefore tempting but, of course, incorrect to interpret (2·1) as simply a probabilistic description which says that the system is actually in one of the

*) Of course it is a widely held view that quantum mechanics should never be applied to single microsystems but only to ensembles of identically prepared microsystems. If one takes this point of view, the language in which the Cat paradox is formulated has to be modified but it loses none of its force. For the sake of brevity I do not give the reformulation here.

states φ_i with probability $|c_i|^2$, and that measurement simply removes our ignorance and therefore the need for a probabilistic description, just as in the standard classical applications of probability theory. A formal way to see that this interpretation is indeed incorrect is to write down the density matrices corresponding to the two cases: the description given in the last sentence requires the density matrix to be of the "mixture" form

$$(\rho_{ij})_{\text{mixture}} = |c_i|^2 \delta_{ij} \tag{2·2a}$$

while the pure state (2·1) has the density matrix

$$(\rho_{ij})_{\text{pure}} = c_i{}^* c_j \,. \tag{2·2b}$$

Although (2·2a) and (2·2b) give identical predictions for the probability of obtaining the various eigenvalues a_i of A on measurement, they give quite different predictions for measurement of operators which do not commute with A. (For example, in our spin $\frac{1}{2}$ case, if we put $c_\uparrow = c_\downarrow = 2^{-1/2}$ and measure S_x, Eq. (2·2a) predicts equal probabilities for obtaining the values $\pm\frac{1}{2}\hbar$, whereas (2·2b) predicts that we are certain to obtain the eigenvalue $+\frac{1}{2}\hbar$.) Thus, a system in a pure state does not "have" a definite value of any operator which we may choose to measure on it. On the other hand, the standard measurement axioms tell us that immediately a measurement of A is made the description changes discontinuously: If we look at the result and find the value a_k, then the density matrix thereafter is just $\rho_{ij} = \delta_{ik}\delta_{jk}$ (corresponding to $\psi \to \varphi_k$ as above), while if we make the measurement but do not look at the result, it is of the form (2·2a) and hence admits a classical probabilistic interpretation in which the system does "have" a particular value of a_i but we do not know it.

So far, so good; the slight unease that students of physics often feel at this state of affairs is usually exorcized by teachers and textbooks by invoking Bohr's dictum that microscopic systems should not even be thought of as possessing properties in their own right in the absence of the (macroscopic) experimental conditions. Once we have set up our measuring apparatus to measure (say) S_z (runs the argument) we have ipso facto excluded the possibility of measuring S_x for the same system at the same time, and so we may if we wish describe our system by the "mixture" density matrix (2·2b) even before the measurement actually takes place without fear of contradiction by experiment. To the objection that we could have chosen to measure S_x instead, the reply is that since the experimental conditions would then be different, it is illegitimate to assume that the description of the microsystem must remain the same.

Provided that one is content to make, with Bohr, a sharp distinction between microsystems and the macroscopic measuring apparatus—or (equivalently?) to accept the notion of "measurement" as an operation which cannot be defined within the framework of quantum mechanics itself—it is hard to fault this line of reasoning; see for example the philosophically sophisticated version given by Reichenbach.[5] The real trouble only starts when we take seriously the fact that the measuring apparatus[*)]—be it a photographic plate, a Geiger counter, the human retina or anything else—is itself a physical system made up of atoms and electrons and therefore should in principle be describable in quantum-mechanical terms. It should therefore be legitimate to ask what happens if instead of treating "measurement" as something quite extraneous to the ordinary behaviour of physical systems, we treat it as merely a particular type of physical process and describe it by the linear time-dependent Schrödinger equation. Consider then an apparatus set up to measure on a given set of microsystems the value of an observable A which, as above, possesses eigenfunctions φ_i and (nondegenerate) eigenvalues a_i. Oversimplifying somewhat (for the complications which occur in a realistic description, see Ref. 4)), let us suppose that initially the apparatus is in some quantum state X_0, and consider first the case in which the initial state of the microsystem is the eigenfunction φ_i of A. Since we wish to "read off" the value of A from the final state of the apparatus, it follows that the result of the interaction between microsystem and apparatus must be to throw the latter into some final state X_i, which is in general different from X_0. Moreover, if we are to be able to read off the value of a_i unambiguously, the states X_i corresponding to different values of i must be orthogonal. Thus, we write the effect of the interaction symbolically in the form (for an "ideal" measurement[4)]).

$$\varphi_i X_0 \xrightarrow{\text{int}} \varphi_i X_i \qquad (2\cdot 3)$$

or more formally

$$\hat{U}\,(t\to\infty)\,\varphi_i X_0 = \varphi_i X_i\,, \qquad (2\cdot 4)$$

where $\hat{U}\,(t) \equiv \exp i\hat{H}t$ is the time evolution operator of the "universe" (microsystem plus apparatus). A further requirement on the states X_i is that they should be not only mutually orthogonal but macroscopically distinguishable. A detailed discussion of the way in which some familiar types of measuring apparatus fulfil these requirements (and others, see below) can be found in Refs. 6) and 7). So far, there is no particular paradox: For any given value of a_i the apparatus ends up in the corresponding state X_i in which it possesses definite macroscopic properties.

Now, however, consider the case in which the initial state of the microsystem was a *linear superposition* $\psi = \sum_i c_i \varphi_i$. *If* we assume that the time-

*) More precisely, that part of it which detects and records the behaviour of the microsystem. In a Stern-Gerlach experiment, for example, the deflecting magnet is essentially irrelevant in the present context but the photographic plate is crucial.

dependent Schrödinger equation does indeed correctly describe the interaction between microsystem and apparatus, then the linearity of the time evolution operator $\hat{U}(t)$ forces us to conclude that in this case the effect of the interaction is

$$(\sum_i c_i\varphi_i)X_0 \xrightarrow{\text{int}} \sum_i c_i\varphi_i X_i\,. \tag{2·5}$$

Thus, under these conditions the macroscopic apparatus, and more generally any part of the macro-world which has suffered changes in the course of the measurement process, does not end up in a state with definite macroscopic properties at all—a state of affairs which is often summed up picturesquely and somewhat inaccurately by saying that "Schrödinger's Cat ends up in a linear superposition of states of being dead and being alive". Whatever assumptions we make about the description of subsequent "measurements", etc., such a conclusion clearly conflicts prima facie with our most basic common-sense conceptions about the behaviour of the macroscopic world.

In many discussions of the Cat paradox, and in the subsequent considerations of the present paper, a crucial role is played by the following trivially demonstrable theorem. Consider two interacting systems 1 and 2 each with a complete set of orthonormal eigenfunctions $\{\varphi_i(1)\}$, $\{\chi_j(2)\}$ respectively. Then the pure state

$$\psi=\sum_i c_i\varphi_i(1)\chi_i(2) \tag{2·6}$$

cannot be distinguished from a mixture of the states $\varphi_i(1)\chi_i(2)$ with probability $|c_i|^2$ by any measurement carried out only on system 1 or only on system 2. It can be distinguished from the mixture by measuring correlations of the form $\langle\hat{B}(1)\hat{C}(2)\rangle$ where $\hat{B}$ operates on system 1 and $\hat{C}$ on system 2, but only provided (at least) that $\hat{B}$ and $\hat{C}$ are both nondiagonal in the representation labelled by the φ_i and χ_i. It is a very standard application of this theorem to the quantum theory of measurement to observe that the final state on the right-hand side of (2·5) is equivalent to a mixture as regards any subsequent measurements carried out on the microsystem by itself, and this observation is often regarded as a justification of the so-called projection postulate. However, for present purposes it is the state of the macroscopic apparatus which is of prime interest. Let us therefore from now on forget about the microsystem*) and treat the apparatus as itself possessing a pure-state wave function which is a linear superposition of macroscopically different states:

$$\psi_{\text{app}}=\sum_i c_i X_i\,. \quad (X_i, X_j)=\delta_{ij} \tag{2·7}$$

*) This is actually a quite realistic description of some highly non-ideal measurements, in particular the measurement of photon polarization. In this case the photon is usually absorbed, so the final state of the "universe" is indeed of the form (2·7).

Much discussion in the quantum theory of measurement has centred around the question: Is there anything intrinsically absurd about the description of a macroscopic body by a state of the form (2·7)? It is necessary to review briefly one or two aspects of the discussion which are relevant to the present paper.

Clearly the description (2·7) would be, if not absurd, at least very surprising, if it were possible actually to demonstrate the occurrence of interference between the macroscopically different states X_i. Therefore, much effort has gone into showing that this is impossible, that is that the pure state (2·7) gives precisely the same predictions for all realistically observable quantities as a mixture of the states X_i with weight $|c_i|^2$. (Whether this remark, even if true, solves the "quantum measurement problem" is a deep question with philosophical as well as physical aspects. A substantial minority of physicists, including the present author, feels it does not. See e.g. Ref. 8).) A central theme in the argument may be put crudely as follows: The measuring apparatus (or any macroscopic object) is a very complex assembly of atoms, electrons, etc., and if the final states X_i, X_j are to be recognizably (macroscopically) different, then crudely speaking a large number of atoms must be behaving differently in the two states. Purely to make the exposition simple, let us assume that in the initial state a large number of atoms, say N, were in some single-particle state χ_0, and that in the macroscopic state i these have all made the transition into a different single-particle state χ_i, where $(\chi_i, \chi_j) = \delta_{ij}$. Then, schematically, the final wave function in (2·7) is of the form

$$\psi_{\mathrm{app}} = \psi_0 \sum_i c_i \chi_i(1)\, \chi_i(2) \cdots \chi_i(N)\,, \tag{2·8}$$

where ψ_0 describes the atoms whose state is unchanged. By an obvious extension of the theorem quoted above, to distinguish (2·8) from a mixture would require us to measure (at least) the expectation value of an N-particle operator of the form ($\hat{B}, \hat{C}, \cdots \hat{Z}$ need not of course all be different)

$$\hat{\Omega} \equiv \hat{B}(1)\, \hat{C}(2) \cdots \hat{Z}(N)\,, \tag{2·9}$$

where $\langle \chi_i(1) | \hat{B}(1) | \chi_j(1) \rangle \neq 0, \cdots \langle \chi_i(N) | \hat{Z}(N) | \chi_j(N) \rangle \neq 0$ for some $i \neq j$. No measurement of any correlation between less than N particles will distinguish the pure state (2·8) from a mixture. This general conclusion is qualitatively independent of the specific model of the measuring apparatus. It is a recurring theme in discussions of the quantum theory of measurement that such measurements are in practice impossible; in particular, even if immediately after the act of measurement N is small enough to make it feasible, the irreversible interactions taking place subsequently in the apparatus (or between the apparatus and its environment) will rapidly make N so large that such a measurement would be totally out of the question. In fact, it is often argued (Refs. 6), 9)) that such measurements *must* be impossible if the result of the measurement is to be thermodynamically stable.

Clearly the argument as to whether the pure state (2·7) is or is not distinguishable from a mixture is only of relevance if one believes that (2·7) is the correct description in the first place. But this description only follows under the assumption that the linear laws of quantum mechanics can be applied strictly to any physical system, however macroscopic and complex. This assumption is not a trivially obvious one; it would, for example, not necessarily be a priori absurd to postulate that at a certain level of complexity non-linear terms begin to play a role and cause superpositions of the form (2·7) to evolve continuously into one of their branches (cf., e.g., Ref. 10)). Let us therefore ask: What *experimental* evidence do we or could we have that states of the form (2·7) actually occur in nature? It is to this question that the rest of this paper is addressed.

§ 3. Disconnectivity and macroscopic quantum phenomena

To discuss the question just posed it is convenient to introduce a semi-quantitative measure of the property which is characteristic of states of the form (2·7); we call this the "disconnectivity" (D). Crudely speaking, we want D to be a measure of the subtlety of the correlations we need to measure to distinguish a linear superposition from a mixture. A variety of different quantitative measures will fulfil this role; for the purpose of the present paper (though quite possibly not more generally) the following seems to be adequate. We consider for the sake of simplicity of exposition a system of N' identical bosons (some refinements in the definition are necessary for the case of fermions or distinguishable particles, but these do not affect the main thrust of the argument). Then for any integer $N \leq N'$ we can define the reduced density matrix ρ_N, expressed in an appropriate basis of products of one-particle states: e.g. in the coordinate representation we have

$$\rho_N(\boldsymbol{r}_1, \boldsymbol{r}_2, \cdots, \boldsymbol{r}_N : \boldsymbol{r}_1', \boldsymbol{r}_2' \cdots, \boldsymbol{r}_N') \equiv \int d\boldsymbol{r}_{N+1} \cdots d\boldsymbol{r}_{N'} \Psi(\boldsymbol{r}_1 \boldsymbol{r}_2 \cdots \boldsymbol{r}_N : \boldsymbol{r}_{N+1} \cdots \boldsymbol{r}_{N'}) \times \Psi^*(\boldsymbol{r}_1' \boldsymbol{r}_2' \cdots \boldsymbol{r}_N' : \boldsymbol{r}_{N+1} \cdots \boldsymbol{r}_{N'}) . \tag{3·1}$$

We now introduce the "reduced entropy"

$$S_N \equiv -\mathrm{Tr}\, \rho_N \ln \rho_N \tag{3·2}$$

and the quantity

$$\delta_N \equiv S_N / \min_{(M)} (S_M + S_{N-M}) , \tag{3·3}$$

where in the case that both numerator and denominator are zero δ_N is to be taken equal to 1. δ_1 is taken as 0 by definition. We then define the disconnectivity D of a many-particle state as the largest integer N for which δ_N is smaller than some small fraction a. (For the present purpose the precise value of a is not crucial.)

To illustrate the definition let us apply it to the more or less trivial case $N'=2$, for which the only interesting question is whether D is 1 or 2. Let the one-particle basis for particle 1 be labelled $\{\varphi_i(1)\}$ and that for particle 2 $\{\chi_i(2)\}$ (for identical particles the χ_i are of course some linear combination of the φ_i). Consider three types of state:

(1) product wave function, $\psi(1,2)=\varphi(1)\chi(2)$ ($\varphi=\sum_i c_i\varphi_i$ etc.) Here clearly $S_2=1$, so $D=1$.

(2) statistical mixture of the states $\varphi_i(1)\chi_i(2)$ with probability $|c_i|^2$: Here $S_2=S_1=-\sum_i|c_i|^2\ln|c_i|^2$, so $\delta_2=\frac{1}{2}$ and $D=1$.

(3) linear superposition $\psi(1,2)=\sum_i c_i\varphi_i(1)\chi_i(2)$. Here $S_1=-\sum_i|c_i|^2 \ln|c_i|^2$ but $S_2=0$, so $\delta_2=0$ and $D=2$.

Thus, the property of disconnectivity as defined above enables us to distinguish, at least in a qualitative way, states which possess many-particle correlations which are quantum rather than statistical in nature.*)

It should be emphasized that while the definition of D given above is invariant under unitary transformations of the single-particle basis states, it is *not* in general invariant under transformation to collective coordinates (such as the familiar transformation to centre-of-mass and relative coordinates). It is possible that a more satisfactory definition of D would make it invariant under at least some of these more general transformations, but I shall not discuss this question here.

Turning back now to our original question, we see from Eq. (2·7) that the kinds of state important in the Cat paradox, and more generally in the quantum theory of measurement, always have disconnectivity of order N, where N is a *macroscopic* number. The question now arises whether we have any experimental evidence for the existence of such high-D states in nature? In particular, do the so-called "macroscopic quantum phenomena" in superconductors and superfluids automatically provide such evidence?

The general question is discussed in the next two sections, but it is immediately obvious that the answer to the particular one is no. To see this, we shall simply demonstrate that the accepted explanations of the "macroscopic quantum phenomena" in question in quantum-mechanical terms in no way require the existence of high-D states.

Consider first the case of a simple Bose superfluid such as He II at zero temperature. A simple ansatz for the wave function is that appropriate to a free Bose gas:

*) This approach clearly has much in common with Yang's discussion[11] of the concept of off-diagonal long-range order.

$$\Psi(\boldsymbol{r}_1\boldsymbol{r}_2\cdots\boldsymbol{r}_N)=\varphi(\boldsymbol{r}_1)\varphi(\boldsymbol{r}_2)\cdots\varphi(\boldsymbol{r}_N), \tag{3·4}$$

where $\varphi(\boldsymbol{r})$ is some single-particle wave function (not necessarily the ground-state or even an energy eigenstate). At finite temperature the system has to be described by a density matrix in which a macroscopic fraction of the N particles are still in the state $\varphi(\boldsymbol{r})$ and the rest are distributed over different single-particle states. An ansatz such as (3·4) clearly accounts, for example, for the phenomenon of " nonclassical rotational inertia" (see e.g. Ref. 12)); it cannot of course account for some other phenomena of superfluidity such as the metastability of superfluid flow, and to do this it is necessary to put in interactions between the particles so as to produce a finite compressibility. However, this does not change the situation qualitatively (as we shall see in the next section, similar effects arise even in non-superfluid systems) and the disconnectivity so produced is certainly not of macroscopic order.*) The case of a Fermi superfluid (electrons in superconductors, superfluid ^{3}He-A and B) is not much more complicated. The simplest ansatz for the wave fuction of such a system at $T=0$ is just the particle-conserving version of the BCS ansatz for superconductors:

$$\Psi(\boldsymbol{r}_1\sigma_1, \boldsymbol{r}_2\sigma_2\cdots\boldsymbol{r}_N\sigma_N)=\mathcal{A}\varphi(\boldsymbol{r}_1\sigma_1\boldsymbol{r}_2\sigma_2)\varphi(\boldsymbol{r}_3\sigma_3\boldsymbol{r}_4\sigma_4)\cdots\varphi(\boldsymbol{r}_{N-1}\sigma_{N-1}\boldsymbol{r}_N\sigma_N), \tag{3·5}$$

where $\mathcal{A}$ is the antisymmetrization operator. A wave function of the type (3·5), when applied to the electrons in superconductors, is entirely adequate to account for phenomena such as the Josephson effect, which is widely regarded as the paradigm of a "macroscopic quantum effect"; to do this we ignore the electron spins and choose $\varphi(\boldsymbol{r}_1, \boldsymbol{r}_2)$ to be of the form

$$\varphi(\boldsymbol{r}_1, \boldsymbol{r}_2)=a\varphi_L(\boldsymbol{r}_1, \boldsymbol{r}_2)+b\varphi_R(\boldsymbol{r}_1, \boldsymbol{r}_2), \tag{3·6}$$

where $\varphi_L(\varphi_R)$ is a two-particle wave function in which both electrons are localized on the left (right) side of the Josephson junction. The calculation of the Josephson effect then proceeds e.g. along the lines indicated by Feynman.[13] A similar case, even better suited to our present discussion, is the phenomenon of longitudinal magnetic resonance in ^{3}He-A, which is the "internal" analogue of the Josephson effect;[14] here we neglect the spatial coordinates of the Cooper pairs and write the spin function $\varphi(\sigma_1\sigma_2)$ in the form

$$\varphi(\sigma_1\sigma_2)=\frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle+e^{i\Delta\varphi}|\downarrow\downarrow\rangle). \tag{3·7}$$

It is clear that the disconnectivity of the wave function (3·5), with the choices (3·6) or (3·7) for the Cooper pair function φ, is 2 (we ignore here the slight complications associated with the Fermi statistics). Again, the introduction of corrections to the form of the BCS wave function does not change things qualitatively.

*) Equation (3·4) obviously corresponds to $D=1$.

Consequently, we reach the conclusion that the phenomena in superfluid systems which are conventionally cited as evidence for the validity of quantum mechanics on the macroscopic scale do not in themselves require the introduction of high-D states. To put it another way, the explanation of these phenomena in the standard quantum-mechanical way requires only that one- and two-particle correlations are predicted correctly by Schrödinger's equation; it does *not* require that N-particle correlations are correctly predicted, except in so far as they are factorizable in terms of 1- and 2-particle correlations. To sum up the point crudely and schematically, "macroscopic quantum phenomena" require a many-particle wave function of the form,

$$\psi=(a\varphi_1+b\varphi_2)^N, \tag{3·8}$$

while the states of importance in the quantum theory of measurement are of the form

$$\psi=a\varphi_1{}^N+b\varphi_2{}^N. \tag{3·9}$$

§ 4. Where can we find high-*D* states?

Having convinced ourselves that the familiar "macroscopic quantum phenomena" are no evidence in themselves for high-D states, let us enquire more generally how we might verify the existence of such states. There are a number of obvious possibilities:

(a) Direct tests. The highest value of D whose existence has been *explicitly* established in existing experiments is 2: The experiments designed to test the "nonlocal" aspects of quantum mechanics establish en route the existence of $D=2$ states (since any $D=1$ state of the photon field can be mimicked by an "objective local" theory, and—with minor reservations—the whole class of such theories is ruled out by the experimental results[15]). It might no doubt be possible in principle to test directly for the existence of $D=3, 4\cdots$ states by examining (e.g.) the polarization of photons emitted in multiple cascade processed, but the experimental difficulties of going to large values of N are obvious.

(b) Indirect evidence from many-body systems. In so far as theoretical calculations of properties such as binding energy, thermodynamic functions, neutron scattering characteristics etc. of condensed systems rely on *quantum* (not just statistical!) correlations at the 3-particle and higher level, and these calculations give agreement with the experimentally observed values, this could be claimed as circumstantial evidence for the existence of states with $D>2$. However, it is rare to find that such correlations play an essential role in the calculations; in fact, most of the standard approximation methods of many-body theory involve at one stage or another the factorization of even 3-particle correlations into 2- and 1-particle ones. Certainly I know of no such calculations where correlations of *macroscopic* order play a role, and indeed if they did one would tend to be suspicious on general physical grounds.

(c) Diffraction of complex systems. A more direct piece of evidence for the existence of states which, at least by the definition of the present paper, possess high disconnectivity could be sought in experiments on the diffraction of heavy atoms or molecules. In a (hypothetical) Young's slits experiment on a beam of atoms, the natural interpretation of the occurrence of the standard diffraction pattern would be in terms of a wave function for the atomic centre of mass, $\boldsymbol{R}$, which at an intermediate stage in the passage of the beam through the slits, say at time t_0, corresponds to a linear superposition of two states localized in widely different regions of space (i.e. each near a different one of the two slits):

$$\psi(\boldsymbol{R}, t_0) = a\varphi_1(\boldsymbol{R}, t_0) + b\varphi_2(\boldsymbol{R}, t_0), \quad (\varphi_1, \varphi_2) \approx 0\,. \tag{4·1}$$

Since the A nucleons and the Z electrons in the atom are presumably all localized within an atomic distance of the centre of mass, the explicit form of the wave function is

$$\psi(\boldsymbol{r}_1\boldsymbol{r}_2\cdots\boldsymbol{r}_N: t_0) = a\chi_1(\boldsymbol{r}_1\boldsymbol{r}_2\cdots\boldsymbol{r}_N: t_0) + b\chi_2(\boldsymbol{r}_1\boldsymbol{r}_2\cdots\boldsymbol{r}_N: t_0)$$
$$(N = A + Z) \tag{4·2}$$

with

$$\int d\boldsymbol{r}_i\chi_1^*(\boldsymbol{r}_1\boldsymbol{r}_2\cdots\boldsymbol{r}_N:t_0)\,\chi_2(\boldsymbol{r}_1\boldsymbol{r}_2\cdots\boldsymbol{r}_N:t_0) \approx 0, \quad i = 1, 2\cdots N. \tag{4·3}$$

From these considerations it is obvious that the intermediate state of the system at $t = t_0$ has $D = N$, so that the observation of a diffraction pattern would constitute prima facie evidence for the occurrence of high-D states.

It is interesting to enquire how such a conclusion fits in with the claim made in § 2 that to distinguish such a high-D pure state from an (uninteresting) mixture one would have to measure the correlations of N operators each of which is nondiagonal in the representation of interest. The point is that what one actually measures in a Young's slits experiment is the average intensity falling on some point $\boldsymbol{R}_0$ of the detecting screen at some time $t_1 > t_0$, i.e. the expectation value

$$\langle\delta(\boldsymbol{R} - \boldsymbol{R}_0)\rangle(t_1) \equiv (\psi(\boldsymbol{R}, t_1)\,\delta(\boldsymbol{R} - \boldsymbol{R}_0)\,\psi(\boldsymbol{R}, t_1))\,. \tag{4·4}$$

This is equivalent to a measurement at time t_0 of the operator $\exp i\hat{H}(t_0 - t_1)\,\delta(\boldsymbol{R} - \boldsymbol{R}_0)\exp - i\hat{H}(t_0 - t_1)$. Although $\hat{H}$ itself contains only one- and two-particle operators, this expression obviously contains products of N and more operators which are of just the right form to distinguish the states (4·2) from a mixture.

To the best of my knowledge no Young's-slits experiments have actually been carried out with atoms heavier than He. (A *single*-slit diffraction experiment has been carried out with a beam of K atoms.[16]) It is clear in any case that any attempt to extend such experiments to large N would immediately run into two serious difficulties. One well-known difficulty is that in any diffraction experiment the spacing of the diffraction pattern is proportional to the de Broglie wavelength λ, which for fixed energy is inversely proportional to the square root of the particle mass; since presumably in most realistic experimental situations the energy must be at least comparable to the thermal energy kT, this means that the spacing will rapidly become unobservably small as N (and hence M) increases. (This is the kind of reason usually given in textbooks as to why it is impossible to see quantum-mechanical effects with objects of macroscopic size.) A more subtle and less well-known difficulty is associated with the fact that *any* interaction with the environment which results in a change in the state of the latter which is different for the two possible paths is in effect a "measurement" of which slit the system passed through and results in the destruction of interference; the case of a true measurement discussed in § 2 is just a special case of this phenomenon, which is the essence of the famous "γ-ray microscope" thought-experiment of Heisenberg. Clearly, the probability of such an interaction increases very rapidly with the size and complexity of the system in question (consider for example the excitation of vibrational modes of the system by the black-body radiation field). This general problem is to my mind much more fundamental than the limitation imposed by the de Broglie wavelength, and is likely to impose very severe constraints on any attempt to observe interference between macroscopically different states of a macroscopic body, whether in a true diffraction experiment or elsewhere. (Compare the remarks about quantum coherence in the next section.)

(d) Quantization of classical periodic motion. Let us consider for definiteness a macroscopic system which classically undergoes simple harmonic oscillation—say a simple LC-circuit. If we quantize the motion according to the canonical procedure[17] we get discrete energy levels separated by $\hbar\omega_0$, $\omega_0 = (LC)^{-1/2}$, and we would expect the effect of the quantization to be important when the temperature is low enough that $kT \lesssim \hbar\omega_0$ — that is, $T \lesssim$ a few K for reasonable circuit parameters. Do high-D states play an essential role here, and could their presence be explicitly verified?

There are at least two problems here. In the first place, it is not so trivial even to verify that such a system is behaving quantum-mechanically at all; the response of a quantum harmonic oscillator (more precisely, of the observable quantities such as the expectation value of the coordinate) to an external classical force is identical to that of the corresponding classical system, and experiments on quantization of energy loss in (e.g.) electron scattering, even if feasible, could always be interpreted in terms of quantization only of the *electron* energy levels (cf. the well-known situation concerning the photoelectric effect). However, let us assume that we have somehow verified that the energy levels (and zero-point noise, etc.) are indeed as predicted by quantum mechanics. Unfortunately this by itself still does not tell us anything about the existence or otherwise of high-D states, since the most complex operators involved in producing this conclusion are of the form (taking the case of a mechanical oscillator with macroscopic centre-of-mass coordinate X for definiteness)

$$\widehat{X}^2=\sum_{ij}\widehat{x}_i\widehat{x}_j,\quad \widehat{P}^2=\sum_{ij}\widehat{p}_i\widehat{p}_j \tag{4·5}$$

and the requirement that the matrix elements of these operators should be given correctly requires only the existence of $D=2$ states.*)

One could go on to investigate other possibilities such as the creation of non-quasiclassical harmonic-oscillator states, or the quantization of strongly anharmonic classical motion, in a macroscopic system. I shall not do so here, because it must be becoming increasingly obvious that this kind of experiment (like most of the above ones) would have to look for only quantitative modifications either of classically predicted motion or of the behaviour predicted by a theory with only low-disconnectivity states. Clearly what we need is a situation where we expect *qualitatively* new effects. The obvious suggestion is that we might find this in connection with the (classically unknown) phenomenon of quantum tunnelling through a potential barrier, and it is to this that we devote the next section.

§ 5. Quantum tunnelling in macroscopic systems

In discussing this topic it is essential to distinguish between two radically different phenomena: quantum *tunnelling* between states with macroscopically different properties, and quantum *coherence* between such states. (In the context of the discussion of this section, the first is a necessary but by no means sufficient condition for the second.) Consider an isolated system (not necessarily macroscopic) with some generalized coordinate x which moves in a smooth potential $V(x)$ having a metastable minimum (which we will take by convention to correspond to $V=0$) at $x=A$, and a small oscillation frequency ω_0 around this position; suppose that the potential decreases to zero again at some point B, and that the height of the barrier V_0 is much larger than

*) The groundstate and low excited states as described by the usual quantum formalism do of course in general have a high value of D (precisely what value, depends on its definition and the behaviour of the wave function with respect to the relative coordinates). The point is that there is apparently no way of verifying this directly from the dynamics. However, this point deserves more thought.

$\hbar\omega_0$. Then, irrespective of the detailed behaviour of $V(x)$ at points beyond B, we can define the WKB tunnelling amplitude

$$\Gamma = \text{const}\ \omega_0 \exp - \int_A^B \sqrt{2mV(x)}\,dx/\hbar\,. \tag{5·1}$$

In the case in which there is an infinite region on the far side of the barrier where $V(x) < 0$ (as for example in the standard nuclear alpha-decay problem) application of the standard techniques of quantum mechanics yields the result that if initially the system is localized in the metastable minimum, then at long times the probability of still finding it there decreases as $e^{-\gamma t}$, where $\gamma \sim \omega_0^{-1}\Gamma^2$ is the standard tunnelling probability in WKB theory. Strictly speaking, if no observation has been made of the system up to time t, then the correct description is (schematically) by a linear superposition of the form

$$\psi(t) = e^{-\gamma t/2}\psi_{\text{in}} + (1 - e^{-\gamma t/2})^{1/2}\psi_{\text{out}}\,, \tag{5·2}$$

where ψ_{in} (ψ_{out}) describes a state inside (outside) the barrier. However, in practice it is always adequate to replace this superposition by a mixture; in such a case (e.g. in nuclear α-decay) the coherence between the "in" and "out" states is quite unobservable.*)

In the case where $V(x)$ has a second minimum V_2 close to zero and thereafter increases again to large values (a "double well"), the behaviour is more complicated and depends critically on the precise shape and scale of the potential. In particular, in the special case $|V_2| \lesssim \hbar\Gamma$ we get the possibility of resonance oscillations in which the system moves backwards and forwards between the two wells with frequency $\sim 2\Gamma$; equivalently the groundstate is a doublet with splitting $\sim 2\hbar\Gamma$. To obtain such behaviour it is essential to write the wave function in the form of a linear superposition of (approximately) states localized near A and B respectively:

$$\psi(t) = a\psi_A(t) + b\psi_B(t) \tag{5·3}$$

and all effects are lost if we replace this by a mixture. This is a typical quantum *coherence* effect. Well-known examples of such effects in real physical systems include the inversion level of the NH_3 molecule, the effective exchange Hamiltonian in ferromagnets and solid ^3He, the Josephson effect in superconductors (cf. § 3) and, extending the consideration to many degenerate wells, Bloch waves in metals in the tight-binding approximation.

It is crucial to appreciate that in a very important sense the time-scales for these two phenomena are quite different. Quantum *coherence* requires that the relative phase of the wave function should be preserved over times

*) This bald statement needs some qualification; see for example Ref. 18).

of order Γ^{-1}, which may be very long; any "observation" of the system at time intervals τ short compared with this will destroy the coherence effects and effectively localize the system on one side of the barrier or the other.*) (This phenomenon is well-known in various areas of physics under different names: for a general discussion in the present spirit see Ref. 19).) On the other hand, the characteristic time associated with the phenomenon of quantum *tunnelling* is the "bounce time" τ_b given by

$$\tau_b \equiv \int_A^B \frac{dx}{\sqrt{2V(x)/m}} \sim \omega_0^{-1} . \tag{5·4}$$

We may express the difference intuitively by saying that although the system tries unsuccessfully many times, with a frequency ω_0, to pass the barrier, and on average takes a time at least $\sim\Gamma^{-1}$ before it succeeds, when it does pass it the crucial motion takes a time only of order τ_b. To destroy *tunnelling*, therefore, we need to "observe" the system with a frequency of the order of ω_0 or faster—which makes the criterion for its existence far less stringent. In real life the "observation" of our system (particularly if it is macroscopic) is usually not provided by man but is inherent in its dissipative interactions with its environment (cf. below); thus we would expect that there are many situations in which macroscopic systems show quantum tunnelling without quantum coherence.

Let us now imagine that the system described above is indeed macroscopic, so that the states centred around A and beyond the barrier are macroscopically distinguishable. For definiteness we might as well consider what seems to be the experimentally most promising case,[20),21)] that of a superconducting ring interrupted by a Josephson junction of some kind, (i.e. a SQUID[22)] without the tank circuit). In this case the relevant macroscopic variable is the magnetic flux φ trapped in the ring, and the appropriate "potential" is of the form

$$V(\varphi) = -\frac{i_c\varphi_0}{2\pi}\cos\left(\frac{2\pi\varphi}{\varphi_0}\right) + \frac{(\varphi-\varphi_x)^2}{2L}, \tag{5·5}$$

where φ_x is the externally applied flux, L the self-inductance of the ring, i_c the critical current of the Josephson junction and $\varphi_0 \equiv h/2e$ the flux quantum. When $2\pi L i_c/\varphi_0 > 1$ this potential can have more than one minimum, and in particular for $\varphi_x \approx \varphi_0/2$ we can produce two nearly degenerate minima separated by a barrier of height $\lesssim i_c\varphi_0/\pi$. Since the Hamiltonian also contains a term $\frac{1}{2}C\dot{\varphi}^2$ involving the capacitance of the junction, the latter plays the role of the particle "mass" and, if for the moment we treat the system as isolated, it is straightforward to apply the standard WKB formulae. It should be noted

*) More precisely, the system will tend to a mixture of states on the two sides (with equal weight) with a lifetime of the order of $1/\Gamma^2\tau$.[19)]

that since φ is related to the current i circulating in the ring by $\varphi-\varphi_x=Li$, it is essentially a sum of single-particle operators just as in the mechanical case. For future reference we note that with experimentally realistic constraints on the parameters ($L\gtrsim 5\times 10^{-10}$ henry, $C\gtrsim 10^{-15}$F) the quantity Γ cannot exceed about 10^{10} secs^{-1} unless the separation between the two minima is much less than φ_0, and is in fact very much smaller unless the parameters fall in a very narrow "window".

Any observation of quantum coherence between macroscopically (by an amount$\sim\varphi_0$) different flux states*) would evidently constitute very strong prima facie evidence for the existence of high-D states, and indeed would probably be as near as we are likely to get to a laboratory version of Schrödinger's Cat. However, I believe that in practice such an observation is likely to present serious, though possibly not insuperable, difficulties. The basic difficulty is that for such coherence to "exist" in any reasonably meaningful sense at all (let alone to be realistically detectable!) a minimum condition is that the system be not "observed" over a time at least of the order of the period of the resonance oscillation, and macroscopic systems such as SQUIDS, in strong distinction to atoms, interact so strongly with their environment that this condition is by no means trivial to realise. (In this connection it should be remembered that the "environment" need not be something physically external to the system but can include normal electrons, lattice vibrations and much else.) Although intuitively such an argument would tend to make one sceptical about the observation of quantum coherence in any macroscopic system, there are at least two features peculiar to the SQUID case which might tend to counter it: (1) although a typical period of the resonance oscillation is "macroscopic" (say$\gtrsim 10^{-4}$ sec) it is possible that with very precise control of the ring parameters (or luck!) one could make it much smaller: (2) because of the characteristic "quantum purity" of superconductors at very low temperatures,[21] most of the dissipative mechanisms which normally operate in macroscopic systems are eliminated. So let us try to make this a little more quantitative. Suppose that the basic quantum-mechanical variable, the flux φ, interacts with some "environment" variable A which undergoes classical or quantum fluctuations. Then, crudely speaking, we can say that quantum coherence will be totally lost if the root-mean-square fluctuations (at frequency Γ) of the difference in energy between the two originally degenerate states, which is of order $\varphi_0\langle A\rangle_{\mathrm{rms}}$ in our case, becomes comparable to (say $1/2\pi$ times) the tunnelling energy $\hbar\Gamma$. A roughly equivalent statement for thermal equilibrium conditions is that coherence is lost when the damping γ of the resonance oscillation, multiplied

*) Such an idea has been recently canvassed extensively by A. Widom (see e. g. Ref. 23)). Widom's approach is markedly different from the present one and puts little explicit emphasis on the effect of the environment.

by $kT/\hbar\Gamma$ if this factor is large compared to 1, becomes comparable to $\Gamma/2\pi$. In a real SQUID two obvious sources of such fluctuations are the normal current through the junction and the interaction with the radiation field. If we assume that the so-called resistively shunted junction (RSJ) model is applicable to arbitrary small oscillations of the flux, then the damping due to the normal resistance is simply $1/2CR$ where R is the resistance as measured from the current-voltage characteristic, and hence a minimum condition for coherence would be $R \gtrsim \pi(\Gamma C)^{-1}$ (multiplied by $kT/\hbar\Gamma$ if this is large); this would mean in practice a resistance of the order of at least 1 MΩ even with the most optimistic assumptions. However, it should be emphasized that although the RSJ model is very widely used for all types of junction, neither its microscopic basis nor the experimental evidence for it is beyond doubt, and it could just be that the effective resistance for small flux oscillations is actually very high. The effect of the radiation field (which couples to the system, through the term $\varphi\varphi_x/L$ in Eq. (5·5), about 11 orders of magnitude more strongly than to a typical atom!) depends rather critically on the geometry and environment of the SQUID; for a single ring in free space with dimensions $\sim$1 cm and $L \sim 5 \times 10^{-10}$ henry the condition is roughly $\Gamma < 10^7$ sec^{-1} if $kT \lesssim \hbar\Gamma$ and $\Gamma < 10^8\, T^{-1}$ sec^{-1} (T in K) if $kT \gg \hbar\Gamma$. Needless to say the incorporation of the SQUID in (say) a superconducting cavity resonator will improve the situation here, but in the real-life case one must also consider the fluctuations of the external flux due to the device which is actually to detect the coherence effect (note that we certainly cannot monitor the system continuously in the usual way, as this is the one sure way of destroying the coherence!). Clearly the problem deserves a more detailed consideration, which I hope at some stage to give elsewhere; at the time of writing I am inclined to believe that it will turn out to be impossible in practice (at least in the near future) to see "full-blooded" coherence between states differing by a full flux quantum, but that it may possibly just be feasible to see a similar effect when the two states involved differ by a small fraction of φ_0. At any rate it should be clear from the above that in a macroscopic system such as a SQUID, in strong contrast to most atomic systems, the interaction with the environment is so strong as to change the picture qualitatively, and that all existing experiments on SQUIDS's must have missed the minimum conditions for coherence by many orders of magnitude.

If quantum coherence is problematic from an experimental point of view, what of quantum tunnelling by itself? Here the prospects seem a great deal more favourable. We pointed out, above, that the effect of dissipative interactions with the environment on tunnelling is far less severe than on coherence, and this is confirmed by a detailed calculation[24] recently performed by A.O. Caldeira and the author. Transposed to the case under discussion, our results indicate that the effect of dissipation is to suppress the tunnelling probability by a factor of order $\exp -A(\Delta\varphi)^2/R\hbar$, where $\Delta\varphi$ is the distance in flux space the system must travel under the potential barrier, R is the resistance of the junction as above, and A is a numerical constant of order unity. It may be easily verified that this is qualitatively equivalent to saying that damping multiplies the WKB exponent by a factor of order $(1+\gamma/\omega_0)$. Thus, the effect will be relatively small*) if the small oscillations (in this case, the Josephson plasma resonance) are only weakly damped, in accordance with the qualitative considerations given above. Two recent experiments on just such a system[25),26)] have indeed been interpreted by their authors as possible evidence for flux tunnelling, and while it is not clear to the present author that this is the only possible interpretation, it is very likely that the point will be resolved in the near future.

If quantum tunnelling in a macroscopic system such as this is observed, is it evidence for the existence of high-D states? I believe that qualitatively it is, although this question certainly needs a good deal more thought. The point is that the occurrence of quantum tunnelling seems to require the coherent interference of the centre-of-mass wave function at different points under the barrier; evidence for this is the fact that interactions which "measure" the value of the coordinate under the barrier and thereby destroy this coherent interference, also suppress the tunnelling probability according to the above formula. (See the detailed formulation of the problem in Ref. 24).) Suppose this is right; then the centre-of-mass wave function may be explicitly represented as a linear superposition

$$\psi(x) = \int dx_0 a(x_0)\,\psi_{x_0}(x)\,, \quad \psi_{x_0}(x) \equiv \delta(x-x_0) \tag{5·6}$$

and provided that the wave function of *relative* motion of the various component particles is fairly well localized, the state (5·6) clearly has high disconnectivity (of what order exactly, depends on our precise definitions). So the situation is qualitatively similar to that in a Young's slits diffraction experiment, the difference being that the existence of quantum tunnelling is a "zeroth-order" effect rather than a correction to classically expected behaviour.

§ 6. Conclusion

In this paper we have seen that many-body systems, and in particular superconductors and superfluids, do not automatically test the hypothesis that the linear equations of quantum mechanics can be applied to arbitrarily

*) This phrase is somewhat ambiguous. Although the change of the WKB exponent may be only say 10%, this can easily correspond to a suppression of the tunnelling rate itself of one or two orders of magnitude!

complex systems (in the interesting sense of the question) but that under certain conditions they may at least give some relevant clues to the answer. In particular we have seen that of the various places one may look for such clues, the phenomenon of quantum tunnelling in macroscopic systems is probably the most promising. It is, actually, no coincidence that the single most promising experimental arrangement we have been able to find (the SQUID) involves the use of superconductors, but ironically this has rather little to do with the fact that superconductors display "macroscopic quantum phenomena" in the usual sense of the term; this aspect enters only in that flux quantization and the Josephson coupling in the effective potential (5·5) is essential to provide an energy barrier. The aspect of superconductors which is far more important in the present context is simply that the electromagnetic collective excitations (Josephson plasma resonance, etc.) are much more weakly damped than in normal metals, which in turn is a consequence of the very low entropy (very few excitations) at temperatures far below the superconducting transition temperature. It is this property which makes a SQUID the most promising candidate to date for observing macroscopic quantum tunnelling; if it should ever become possible to observe macroscopic quantum *coherence*, the low entropy and consequent lack of dissipation will be absolutely essential.[21)]

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